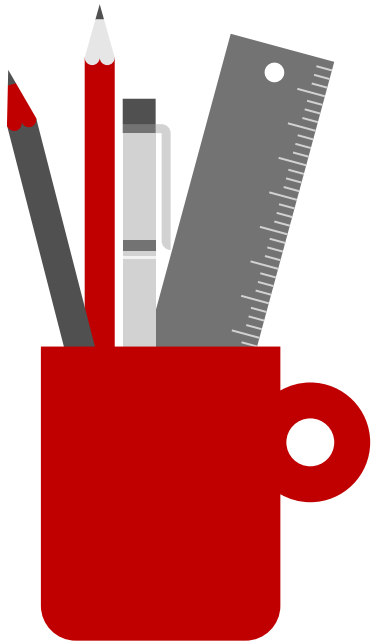

Effective Education Makes Gems out of stones

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Ex. 4.2

i) $a+b=10$, $a-b=6$ $\Rightarrow a^2+b^2=?$

Sol:-

$\therefore 2(a^2+b^2) = (a+b)^2+(a-b)^2$

$\therefore 2(a^2+b^2) = (10)^2+(6)^2$

$2(a^2+b^2) = 100+36$

$a^2+b^2 = \frac{68\cancel{36}}{2}$

$$\boxed{a^2+b^2 = 68}$$

1) ii) $a+b=5$, $a-b=\sqrt{17}$, $ab=?$

Sol:

$\therefore 4ab = (a+b)^2-(a-b)^2$

$4ab = (5)^2-(\sqrt{17})^2$

$4ab = 25-17$

$ab = \frac{8}{4}$

$\boxed{ab=2}$

2) $a^2+b^2+c^2=45$, $a+b+c=-1$, $ab+bc+ca=?$

Sol:-

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$
$$= a^2+b^2+c^2+2(ab+bc+ca)$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$(-1)^2 = 45+2(ab+bc+ca)$$

$$1-45 = 2(ab+bc+ca)$$

$$\frac{-44^{-22}}{2} = \frac{2}{2}(ab+bc+ca)$$

$$\boxed{ab+bc+ca = -22}$$

3) $m+n+p=10$, $mn+np+mp=27$ $\quad m^2+n^2+p^2=?$

Sol:-

$$(m+n+p)^2 = m^2+n^2+p^2+2(mn+np+mp)$$

$$(10)^2 = m^2+n^2+p^2+2(27)$$

$$100 = m^2+n^2+p^2+54$$

$$100-54 = m^2+n^2+p^2$$

$$\boxed{m^2+n^2+p^2 = 46}$$

4) $x^2+y^2+z^2=78$, $xy+yz+zx=59$, $x+y+z=?$

Sol:

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$

$$(x+y+z)^2 = 78+2(59)$$

$$(x+y+z)^2 = 78+118$$

$$\sqrt{(x+y+z)^2} = \sqrt{196}$$

$$\boxed{x+y+z = 14}$$

$$\begin{array}{r|l} 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \\ \hline & 0 \end{array}$$

5) $x+y+z=12$, $x^2+y^2+z^2=64$, $xy+yz+zx=?$

Sol.

$$(x+y+z)^2 = x^2+y^2+z^2+2(xy+yz+zx)$$

$$(12)^2 = 64+2(xy+yz+zx)$$

$$144-64 = 2(xy+yz+zx)$$

$$\frac{80}{2} = xy+yz+zx$$

$$xy+yz+zx = 40$$

6) $x+y=7$, $xy=12$, $x^3+y^3=?$

Sol:-

$\therefore (x+y)^3 = x^3+y^3+3xy(x+y)$

$(x+y)^3 = x^3+y^3+3xy(x+y)$

$(7)^3 = x^3+y^3+3(12)(7)$

$343 = x^3+y^3+252$

$343-252 = x^3+y^3$

$\boxed{x^3+y^3=91}$

7) $3x+4y=11$, $xy=12$, $27x^3+64y^3=?$

Sol:-

$$(3x+4y)^3=(3x)^3+(4y)^3+3(3x)(4y)(3x+4y)$$

$$(11)^3=27x^3+64y^3+36xy(3x+4y)$$

$$1331=27x^3+64y^3+36(12)(11)$$

$$1331=27x^3+64y^3+4752$$

$$1331-4752=27x^3+64y^3$$

$$\boxed{27x^3+64y^3=-3421}$$

8) $x-y=4$, $xy=21$, $x^3-y^3=?$

Sol:-

$\because (x-y)^3 = x^3-y^3-3xy(x-y)$

$(x-y)^3 = x^3-y^3-3xy(x-y)$

$(4)^3 = x^3-y^3-3(21)(4)$

$64 = x^3-y^3-252$

$64+252 = x^3-y^3$

$\therefore x^3-y^3 = 316$

9) $5x-6y=13$, $xy=6$, $125x^3-216y^3=?$

Sol^n:-

$(5x-6y)^3 = (5x)^3-(6y)^3-3(5x)(6y)(5x-6y)$ $\because (x-y)^3 = x^3-y^3-3xy(x-y)$

$(13)^3 = 125x^3-216y^3-90xy(5x-6y)$

$2197 = 125x^3-216y^3-90(6)(13)$

$2197 = 125x^3-216y^3-7020$

$2197+7020 = 125x^3-216y^3$

$\boxed{125x^3-216y^3 = 9217}$

10) $x + \frac{1}{x} = 3$, $x^3 + \frac{1}{x^3} = ?$

$\therefore (a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Sol:-

$$x + \frac{1}{x} = 3$$

$$\left(x + \frac{1}{x}\right)^3 = (3)^3$$

$$x^3 + \frac{1}{x^3} + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) = 27$$

$$x^3 + \frac{1}{x^3} + 3(3) = 27$$

$$x^3 + \frac{1}{x^3} + 9 = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9$$

$$\boxed{x^3 + \frac{1}{x^3} = 18}$$

11) $x - \frac{1}{x} = 7$, $x^3 - \frac{1}{x^3} = ?$

Sol:-

$$\left(x - \frac{1}{x}\right)^3 = (7)^3$$

$$x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) = 343$$

$$x^3 - \frac{1}{x^3} - 3(7) = 343$$

$$x^3 - \frac{1}{x^3} - 21 = 343$$

$$x^3 - \frac{1}{x^3} = 343 + 21$$

$$x^3 - \frac{1}{x^3} = 364$$

12) $3x+\frac{1}{3x}=5$, $27x^3+\frac{1}{27x^3}=?$

Sol:-

$$3x+\frac{1}{3x}=5$$

$$\left(3x+\frac{1}{3x}\right)^3=(5)^3$$

$$27x^3+\frac{1}{27x^3}+3\left(3x\right)\left(\frac{1}{3x}\right)\left(3x+\frac{1}{3x}\right)=125$$

$$27x^3+\frac{1}{27x^3}+3(5)=125$$

$$27x^3+\frac{1}{27x^3}+15=125$$

$$27x^3+\frac{1}{27x^3}=125-15$$

$$27x^3+\frac{1}{27x^3}=110$$

13) $5x - \frac{1}{5x} = 6$, $125x^3 - \frac{1}{125x^3} = ?$

Sol:-

$$\left(5x - \frac{1}{5x}\right)^3 = (6)^3$$

$$(5x)^3 - \left(\frac{1}{5x}\right)^3 - 3(5x)\left(\frac{1}{5x}\right)\left(5x - \frac{1}{5x}\right) = 216$$

$$125x^3 - \frac{1}{125x^3} - 3(6) = 216$$

$$125x^3 - \frac{1}{125x^3} - 18 = 216$$

$$125x^3 - \frac{1}{125x^3} = 216 + 18$$

$$125x^3 - \frac{1}{125x^3} = 234$$

14) $x^3 - y^3 - x + y$ factorize

sol:

$x^3 - y^3 - x + y$ $\quad \because x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$= (x-y)(x^2+xy+y^2) - x + y$

$= (x-y)(x^2+xy+y^2) - 1(x-y)$

$= (x-y)(x^2+xy+y^2-1)$

$=$

14) ii) $8x^3 - \frac{1}{27y^3}$ $\because x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$8x^3 - \frac{1}{27y^3}$

$= (2x)^3 - \left(\frac{1}{3y}\right)^2$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + 2x \times \frac{1}{3y} + \left(\frac{1}{3y}\right)^2\right)$

$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$

15) $(x^2+y^2)(x^4-x^2y^2+y^4)$

$= (x^2+y^2)[(x^2)^2 - x^2y^2 + (y^2)^2]$

$= (x^2)^3 + (y^2)^3$

$= x^6 + y^6$

15) $(x^3 - y^3)(x^6 + x^3y^3 + y^6)$

$\because x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$(x^3 - y^3)((x^3)^2 + x^3y^3 + (y^3)^2)$

$= (x^3)^3 - (y^3)^3$

$= x^9 - y^9$

15) iii) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$

$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)(x^2+y^2)(x^4-x^2y^2+y^4)$

$= (x^3-y^3)(x^3+y^3)(x^6+y^6)$

$= ((x^3)^2-(y^3)^2)(x^6+y^6)$ $\quad \therefore (a-b)(a+b) = a^2-b^2$

$= (x^6-y^6)(x^6+y^6)$

$= (x^6)^2-(y^6)^2$

$= x^{12}-y^{12}$

15) iv) $(2x^2-1)(2x^2+1)(4x^4+2x^2+1)(4x^4-2x^2+1)$

$= (2x^2-1)(4x^4+2x^2+1)(2x^2+1)(4x^4-2x^2+1)$

$= (2x^2-1)[(2x)^2+(2x^2)(1)+(1)^2](2x^2+1)[(2x^2)^2-(2x^2)(1)+(1)^2]$

$= ((2x^2)^3-(1)^3)((2x^2)^3+(1)^3)$

$= (8x^6-1)(8x^6+1)$ $\because (a-b)(a+b)=a^2-b^2$

$= (8x^6)^2-(1)^2$

$= 64x^{12}-1$ Ans.